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The radion contribution to the weak mixing angle

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Abstract

In the Randall-Sundrum (RS) compactification, the gap between two branes is stabilized by the vacuum expectation value of a scalar field called *radion*, ϕ. This radion behaves like a weak interaction singlet scalar field, coupling to matter through the trace of the energy momentum tensor. We find that it can induce a sizable correction to the weak mixing angle if the vacuum expectation value and the mass of the radion is around TeV. We also comment on the contribution to the $K_L - K_S$ mass difference and to $(g-2)_\mu$.

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The recent try of the brane physics on an S^1/Z_2 orbifold model with non-factorizable geometry of space-time [1] has attracted a great deal of attention. It is due to a possibility of generating a large hierarchy of mass scales between two branes, Brane 1 (B1) with a positive cosmological constant(or brane tension) $\Lambda_1 \equiv 6k_1M^3$ and Brane 2 (B2) with a negative cosmological constant $\Lambda_2 \equiv 6k_2M^3$. The bulk between these branes is required to carry a negative bulk cosmological constant $\Lambda_b \equiv -6k^2M^3$. B1 is interpreted as the hidden brane with a fundamental mass scale and B2 is identified with the visible brane. In this setting the metric at B2 has an exponential warp factor which could be used to understand the huge gap between the Planck and eletroweak scales. Although this model introduces cosmological constants k in the bulk and k_1 and k_2 on the branes, it still describes a static universe because of the fine-tuning between the bulk and brane cosmological constants $k = k_1 = -k_2$, which are the consistency conditions in the model.

Hence, if the fine-tuning is not exact, the solution has the time dependence and the universe expands exponentially [2] but its form is not suitable for the standard Big Bang universe after the inflation. To circumvent this cosmological problem, some approximation schemes regarding the brane matter in the static limit has been taken into account and(or) some conditions (such as the positive brane tension) for the brane and bulk cosmological constants are required [3]. With the addition of the Gauss-Bonnet interaction, one can have a finite region of parameter space allowing a positive brane tension at B2 [4]. In Ref. [3], B1 was considered as the visible brane just to circumvent the cosmological difficulty, which contradicts the original motivation for the gauge hierarchy solution [1]. Since this problem can be resolved with the Gauss-Bonnet interaction [4], bulk matter effects and extra dimension stabilization process [5], etc., we consider B2 as the visible brane in this paper.

The simplest example is in the five dimensional world with the fifth dimension denoted as y. Upon compactification, the (yy)-component of the metric tensor (a moduli) behaves like a scalar field which can be light ($\sim$ electroweak scale). This moduli is often called the *radion* since its vacuum expectation value determines the distance scale between B1 and

B2. Since the radion can be light, it may have detectable signatures at the present and future accelerator experiments if the vacuum expectation value of the radion is as small as the electroweak scale [6]. In this scenario, we study the radion contribution to the W and Z masses or to the correction to the weak mixing angle $\sin^2\theta_w$. In addition, we comment briefly on the radion contribution to the other parameters of the electroweak physics, but they do not give very strong constraints.

The fields fluctuating near the RS background are given as

$$ds^2 = e^{-2kb(x)|y|} g_{\mu\nu}(x) dx^\mu dx^\nu - b^2(x) dy^2, \tag{1}$$

where $g_{\mu\nu}$ is the four-dimensional graviton and $T(x)$ is the modulus field. The S^1/Z_2 symmetry forbids the gravi-photons $g_{\mu 5}$. After the Kaluza-Klein (KK) reduction for the massless modes, the 5-dimensional Hilbert-Einstein action gives [7]

$$S = \frac{M^3}{2k}\int d^4x\sqrt{-g}\left(1 - e^{-2kb(x)}\right)R + \frac{3M^3}{k}\int d^4x\sqrt{-g}\partial_\mu\left(e^{-kb(x)}\right)\partial^\mu\left(e^{-kb(x)}\right)\ , \tag{2}$$

where R is the 4-dimensional Ricci scalar. Let us take field definition as follows,

$$\phi(x) = \langle\phi\rangle + \varphi(x) \equiv t\ e^{-kb(x)} \quad \text{with} \quad t = \sqrt{6M^3/k}\ , \tag{3}$$

where $\langle\phi\rangle$ is a vacuum expectation value (VEV) of $\phi(x)$ and $\varphi(x)$ is its fluctuation near the VEV. Note that the t has mass dimension of the Planck scale. Then, we arrive at

$$S = \frac{2M^3}{k}\int d^4x\sqrt{-g}\left(1 - (\langle\phi\rangle + \varphi)^2/t^2\right)R + \frac{1}{2}\int d^4x\sqrt{-g}\partial_\mu\varphi\partial^\mu\varphi\ , \tag{4}$$

from which we see the modulus field $b(x)$ can be interpreted as a 4-dimensional scalar field. We will call the scalar φ "radion" [6]. The radion is basically massless because it is associated with the metric component. But it could get mass and vacuum expectation value after KK reduction. For example, the Goldberger-Wise mechanism [8] or a gaugino condensation in supergravity models would render these to the radion. Therefore, we will assume its mass and vacuum expectation value below.

To obtain radion's phenomenological constraints, one must derive its couplings to gauge bosons and to fermions. The kinetic energy terms of the vector boson and fermion fields, which live on the visible ($y = 1/2$) brane, read

$$\mathcal{L} = e\Big[-\frac{1}{4} g^{MO} g^{NP} F_{MN} F_{OP} + \frac{i}{2}\left(\overline{\psi}\gamma^{\mu} e_{\mu}^{M} \nabla_M \psi - e_{\mu}^{M} (\nabla_M \overline{\psi})\gamma^{\mu}\psi\right)\Big] \,, \tag{5}$$

where ∇ denotes the covariant derivative on a curved manifold, and e_{μ}^{M} and e are the inverse and determinant of the vierbein, respectively. Here M, N indicate generally Einstein indices of the curved space and μ is the Lorentz index of the local reference frame, but in our case the distiction between them is meaningless because our space is conformally flat. Under the transformation

$$\begin{aligned} e_M^{\mu} &\longrightarrow e^{-kb} e_M^{\mu} \approx e^{-kb} \delta_M^{\mu}, \qquad \text{or} \\ g_{MN} &\longrightarrow e^{-2kb(x)} \eta_{\mu\nu} e_M^{\mu} e_N^{\nu} \approx e^{-2kb(x)} \eta_{\mu\nu} \delta_M^{\mu} \delta_N^{\nu} \ , \end{aligned} \tag{6}$$

the kinetic energy term of gauge bosons remain as the canonical form and hence the gauge boson field V_μ is invariant under the above transformation,

$$V_\mu \longrightarrow V_\mu \ . \tag{7}$$

On the other hand, for the fermion field, we should take

$$\psi \longrightarrow \left(\frac{t}{\langle\phi\rangle}\right)^{3/2} \psi \tag{8}$$

to make its kinetic term be canonical form. After redefinition of the fields, the original kinetic terms become

$$\mathcal{L} \ \supset -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \left(1 + \varphi/\langle\phi\rangle\right)^3 i \ \overline{\psi}\gamma^{\mu}\partial_{\mu}\psi \ . \tag{9}$$

Next, let us consider the mass and general gauge interaction terms. With Eqs. (3), (6), (7) and (8), we obtain

$$-\mathcal{L} \ \supset e\Big[-\frac{1}{2} M_V^2 \ V_M V^M + m_f \overline{\psi}\psi + g\overline{\psi}\gamma^{\mu} e_{\mu}^{M} \psi V_M \Big]$$

$$\longrightarrow \; -\frac{1}{2}\Big(1+\varphi/\langle\phi\rangle\Big)^2\Big(M_V\langle\phi\rangle/t\Big)^2\,V_\mu V^\mu + \Big(1+\varphi/\langle\phi\rangle\Big)^4\Big(m_f\langle\phi\rangle/t\Big)\,\overline{\psi}\psi \tag{10}$$

$$+\Big(1+\varphi/\langle\phi\rangle\Big)^3\,g\overline{\psi}\gamma^\mu\psi V_\mu \tag{11}$$

$$= -\frac{1}{2}\Big(1+2\varphi/\langle\phi\rangle\Big)\Big(M_V\langle\phi\rangle/t\Big)^2\,V_\mu V^\mu + \Big(1+4\varphi/\langle\phi\rangle\Big)\Big(m_f\langle\phi\rangle/t\Big)\,\overline{\psi}\psi \tag{12}$$

$$+\Big(1+3\varphi/\langle\phi\rangle\Big)\,g\overline{\psi}\gamma^\mu\psi V_\mu + \cdots \quad . \tag{13}$$

If the parameters M_V and m_f are the Planck scales and $\langle\phi\rangle$ is TeV one, we could get TeV scale vector boson and fermion masses. We can show also that a scalar mass is suppressed by the factor $\langle\phi\rangle/t$ through the similar procedure [1], which scenario is suggested by Randall and Sundrum first as a possible solution to the gauge hierarchy problem. However, from Eqs. (12) and (13), we can see that a low scale $\langle\phi\rangle$ possibly affects electroweak physics significantly by the radion interactions which in turn gives constraints on the mass of the radion.

The low scale $\langle\phi\rangle$ can have observable effects at low energy. The dominant contribution comes at loop orders. Since this theory is not renormalizable, we introduce a cutoff Λ in the Feynman loop integral. If the soft supersymmetry breaking is introduced, Λ can be interpreted as the soft breaking scale. However, the original Randall-Sundrum scenario for a gauge hierarchy solution does not need supersymmetry and hence our Λ is not necessarily linked to the soft supersymmetry breaking scale. Therefore, we simply treat Λ as an input parameter in this paper.

The loop integral generally involves the particle masses in the loop. Hence, if W and Z bosons are in the loop, their ratio is not proportional to their tree level value. The mass ratio of W and Z bosons determines the weak mixing angle. Experimentally, the weak mixing angle is measured very precisely with errors of order $O(10^{-3})$. Also, the radion can contribute to the $\Delta S = 2$ process, but it turns out to be negligible. Figs. 1–4 show the typical contributions of the radion in some relevant low energy processes.

The Fermi constant is defined by the muon decay rate, $\mu^- \rightarrow e^-\nu_\mu\bar{\nu}_e$, and the weak Z boson coupling is determined by the weak neutral current experiments such as $\nu_e e^- \rightarrow \nu_e e^-$.

With the radion, viz. Figs. 1 and 2, the couplings of the charged (W) and neutral (Z) gauge bosons are modified. Then, one can consider a new mass parameter,

$$M'^2_{W,Z} = \frac{M^2_{W,Z}}{1 + (\alpha_{W,Z}/\langle\phi\rangle^2)} \tag{14}$$

where $\alpha_{W,Z}$ denotes the corrections by the radion intercations

$$\begin{aligned} \alpha_{W,Z} &\equiv \frac{9M^2_{W,Z}}{i}\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2 - M^2_{W,Z}}\frac{1}{k^2 - m^2_\phi} \\ &= \frac{9M^2_{W,Z}}{16\pi^2}\int_0^1 dx\Big[\ln\left(\frac{\Lambda^2 + M^2_{W,Z} + x(m^2_\phi - M^2_{W,Z})}{M^2_{W,Z} + x(m^2_\phi - M^2_{W,Z})}\right) - \frac{\Lambda^2}{\Lambda^2 + M^2_{W,Z} + x(m^2_\phi - M^2_{W,Z})}\Big] \\ &= \frac{9M^2_{W,Z}}{16\pi^2}\Big[\frac{m^2_\phi}{m^2_\phi - M^2_{W,Z}}\ln\left(1 + \frac{\Lambda^2}{m^2_\phi}\right) - \frac{M^2_{W,Z}}{m^2_\phi - M^2_{W,Z}}\ln\left(1 + \frac{\Lambda^2}{M^2_{W,Z}}\right)\Big] \ . \end{aligned} \tag{15}$$

where Λ and m_ϕ denote the cuttoff scale and the radion mass, respectively. Of course, $\alpha_{W,Z}$ must be smaller than $\langle\phi\rangle^2$ in Eq. (14) to maintain the validity of perturbativion. Note that $\alpha_{W,Z}$ *are monotonically decreasing functions as* m_ϕ *increase and* $\alpha_Z > \alpha_W > 0$ *always*. Especially, *in the* $m_\phi \gg M_{W,Z}$ *limit, they decrease very slowly because they become purely logarithmic functions in that limit*. Note that a small $\alpha_{W,Z}$ is possible in the region $\langle\phi\rangle^2 \gg M^2_{W,Z}$, $m^2_\phi \gg \Lambda^2, M^2_{W,Z}$ or in the region $\Lambda^2 \ll m_\phi, M^2_{W,Z}$. But the last case makes our "effective" theory inconsistent because particles heavier than a cutoff scale are decoupled.

We can neglect the tree level contribution of the radion (Fig. 3), since the couplings are given by $m_f/\langle\phi\rangle$ (viz. Eq. (12)). Especially, they are irrelevant in the experiments with *light* leptons.

The modified effective masses of the gauge bosons could affect the determination of the weak mixing angle. With the above relations, Eqs. (14), the weak mixing angle θ'_w is given by

$$\cos^2\theta'_w \equiv \frac{M'^2_W}{M'^2_Z} = \frac{M^2_W}{M^2_Z}\cdot\frac{1 + \alpha_Z/\langle\phi\rangle^2}{1 + \alpha_W/\langle\phi\rangle^2} = \cos^2\theta_w\frac{1 + \alpha_Z/\langle\phi\rangle^2}{1 + \alpha_W/\langle\phi\rangle^2} \ , \tag{16}$$

It can be expressed as

$$\left|\frac{\Delta\sin^2\theta_w}{1 - \sin^2\theta_w}\right| \ = \ \frac{\alpha_Z - \alpha_W}{\langle\phi\rangle^2 + \alpha_W} < \delta \ , \quad \text{or} \quad \langle\phi\rangle^2 \geq \frac{1}{\delta}\Big(\alpha_Z - \alpha_W\Big) - \alpha_W \tag{17}$$

where $\Delta \sin^2 \theta_w$ means $(\sin^2 \theta_w^{'} - \sin^2 \theta_w)$ and δ is the uncertainty. Since $\alpha_{W,Z}$ are very slowly decreasing functions in the region $m_\phi \gg M_{W,Z}$, the bound we obtain is valid over a large region.

In Figs. 6 and 7, we show the allowed regions of $\langle\phi\rangle$ and m_ϕ for $\Lambda = 1, 10$ TeV, respectively. Here, we take the recent world average of $\sin^2 \theta_w = 0.23124 \pm 0.00024$ [9]. From these, we note that for $\langle\phi\rangle \simeq 1 - 1.5$ TeV the weak mixing angle does not give a strong constraint at present. However, for low values of $\langle\phi\rangle$ around < 500 GeV, the present value of the uncertainty in the weak mixing angle gives a strong constraint. In these figures, the shaded region is excluded by the 2 standard deviations and the dotted line corresponds to the boundary of one standard deviation.

There can be the radion contributions to the other parameters of the low energy phenomena such as $(g-2)_\mu$ and $\Delta S = 2$ processes. We comment on these briefly. The present difference between theoretical value and experimental value is negligible, $\frac{1}{2}(g-2)_\mu^{exp} - \frac{1}{2}(g-2)_\mu^{SM} = (50.5 \pm 46.5) \times 10^{-10}$ [10]. The radion contribution to $(g-2)_\mu$ is shown in Fig. 5, from which we estimate

$$\frac{1}{2}(g-2)_\mu = \frac{m_\mu^4}{4\pi^2 \langle\phi\rangle^2 m_\phi^2} \tag{18}$$

for $m_\phi \gg m_\mu$, and

$$\frac{1}{2}(g-2)_\mu = \frac{m_\mu^2}{4\pi^2 \langle\phi\rangle^2} \tag{19}$$

for $m_\phi \ll m_\mu$. From these, we obtain $m_\phi \langle\phi\rangle \geq 20$ GeV^2 for $m_\phi \gg m_\mu$, and $\langle\phi\rangle \geq 200$ GeV for $m_\phi \ll m_\mu$. We note that the radion contribution is negligible in $(g-2)_\mu$.

The radion can contribute to the $\Delta S = 2$ process also at the two loop level as shown in Fig. 5. Our estimate contributed by the radion is

$$iT(\bar{K^0} \to K^0) = iT_{SM}\left[1 + \frac{1}{16\pi^2}\frac{M_W^2}{\langle\phi\rangle^2} \times (\text{log. div. terms})\right] \tag{20}$$

where T_{SM} is the SM contribution. The data on 2ReT is $(3.489 \pm 0.009) \times 10^{-12}$ MeV [9]. Thus, we obtain $\langle\phi\rangle \geq 160$ GeV.

In conclusion, it is pointed out that for a low cutoff scale Λ the most significant radion effects can be found in the weak mixing angle if the vacuum expectation value and mass of the radion are below TeV scale.

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FIGURES

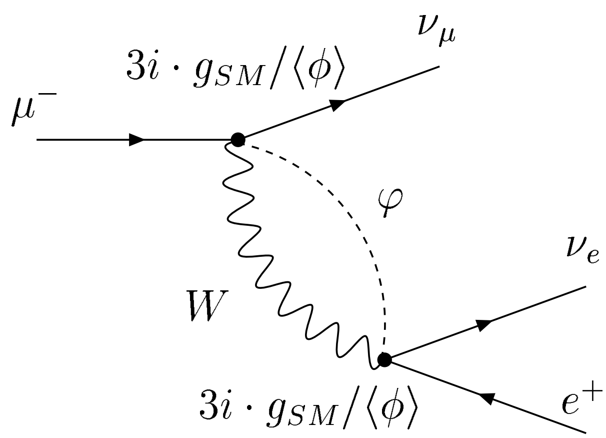

Fig. 1. The muon decay diagram corrected by the radion. g_{SM}'s are the corresponding Standard Model coupling constants in the absence of the radion mediation. We omit the subscripts in g_{SM}.

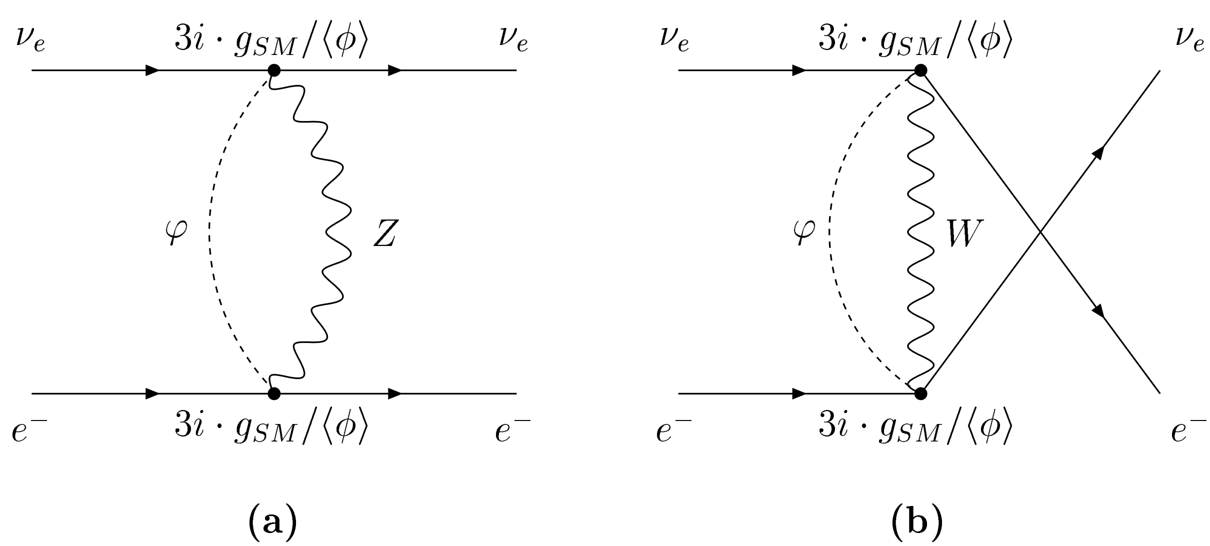

Fig. 2. The radion's contributions to $\nu_e e^- \to \nu_e e^-$. (a) and (b) corresponds to the neutral and charged currents, respectively. g_{SM}'s denote the corresponding Standard Model coupling constants in the absence of the radion mediation.

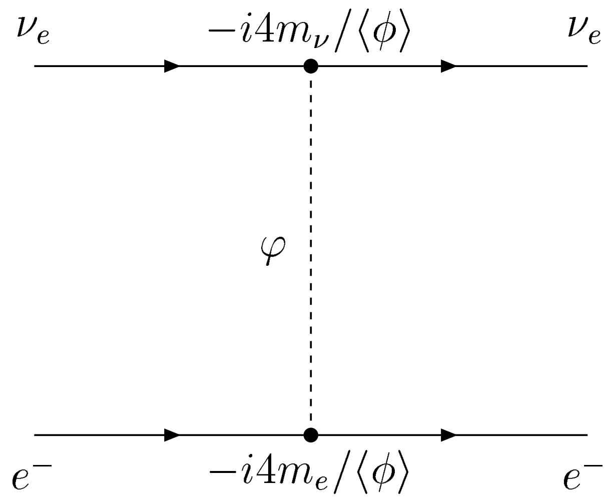

Fig. 3. The radion tree level contribution to $\nu_e e^- \to \nu_e e^-$. It is negligible because of the small couplings.

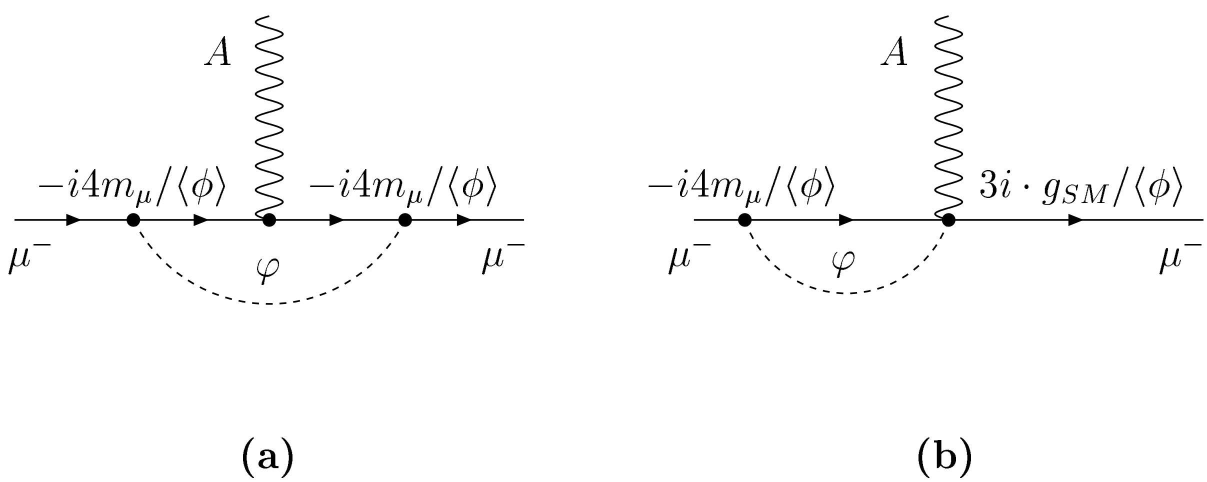

Fig. 4. The muon magnetic moment generated by the radion.

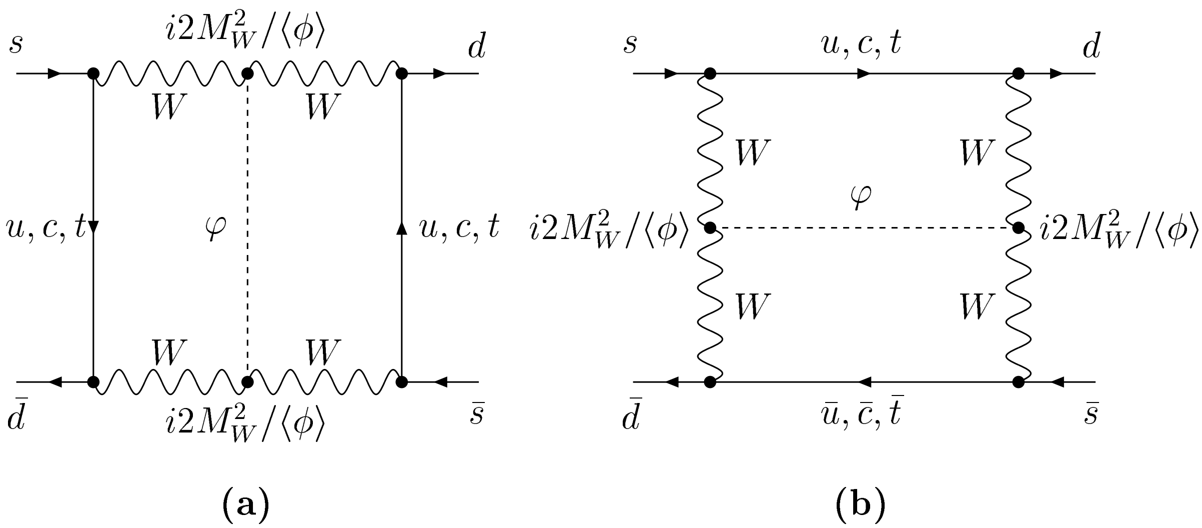

Fig. 5. Dominant radion contributions to the $\Delta S = 2$ process.

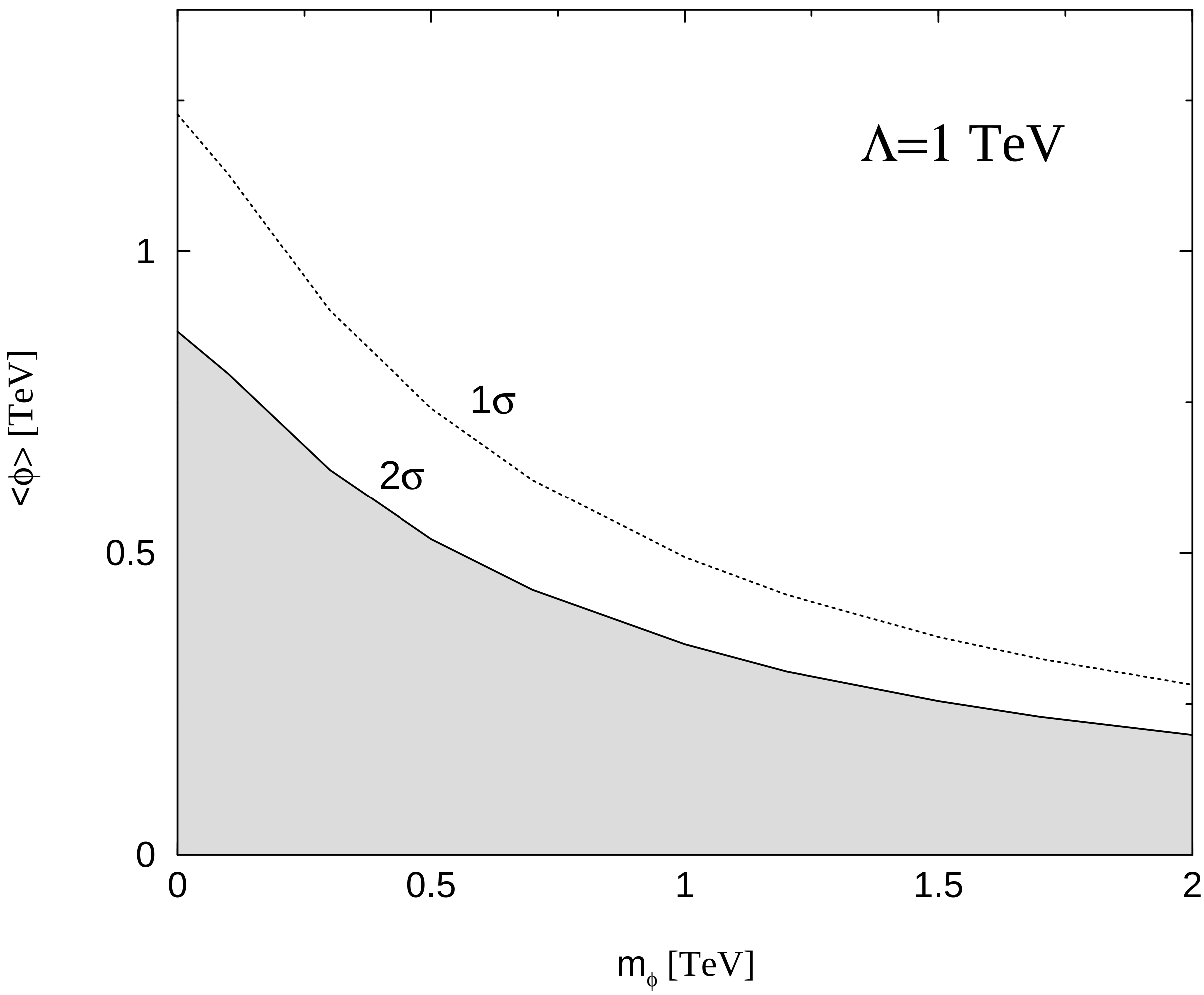

Fig. 6. The allowed region for the vacuum expectation value and mass of the radion from the neutral current data. The cutoff scale is 1 TeV.

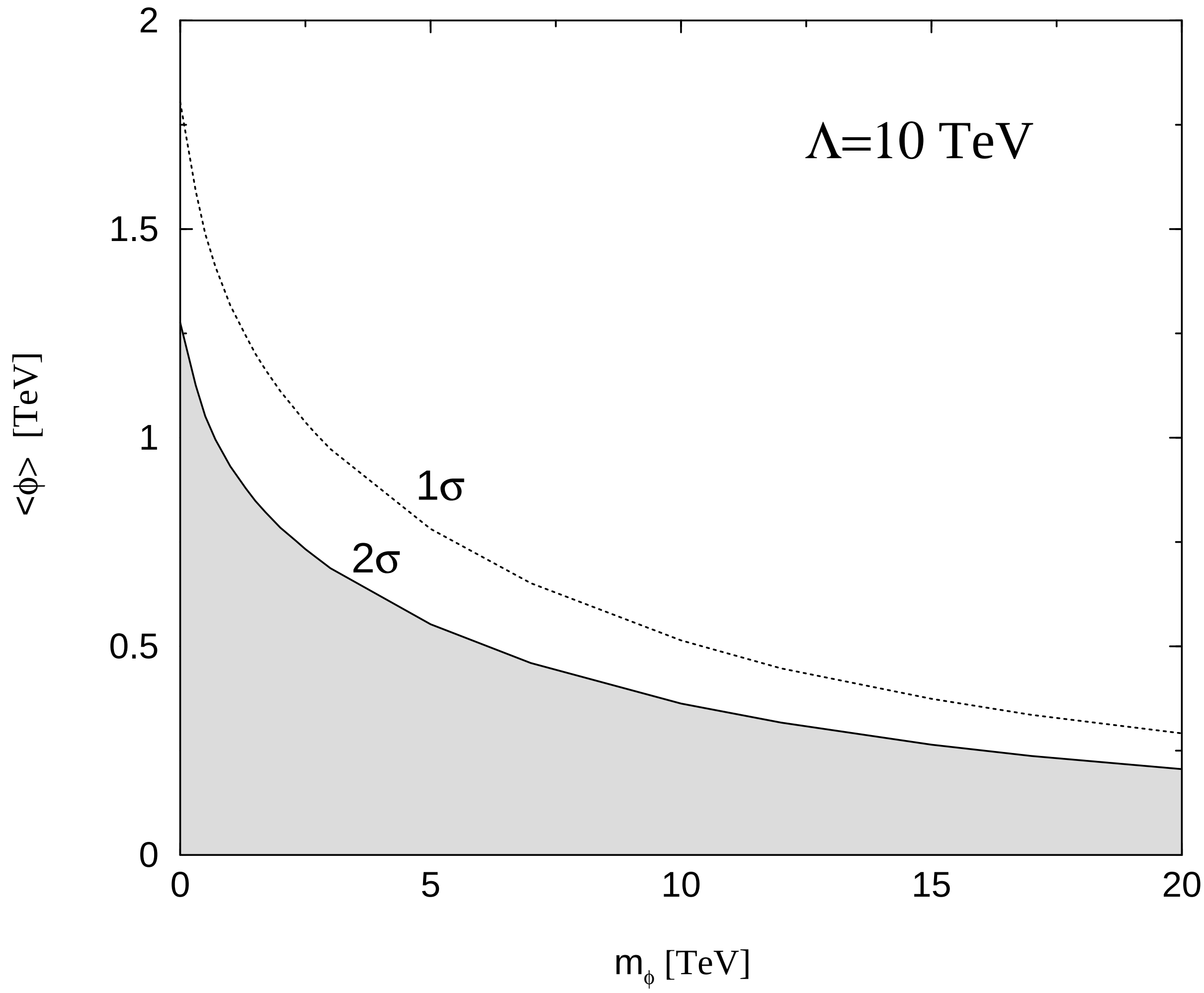

Fig. 7. The allowed region for the vacuum expectation value and mass of the radion from the neutral current data. The cutoff scale is 10 TeV.